Indian Statistical Institute, Bangalore Centre
B.Math. (II Year) : 2009-2010
Semester II : Mid-semestral Examination
Optimization

24.02.2010 Time:2hours. Maximum Marks : 80

Note: The paper carries 83 marks. Any score above 80 will be taken as 80.

1. [16 marks] Let A be an $(m \times n)$ matrix with rank$(A) = m \leq n$. Consider the LPS: *Minimize* $(c^T x)$ subject to $Ax = b, x \geq 0$. (Here $c \in I\!R^n, b \in I\!R^m$.). Suppose $\tilde{x} \in I\!R^n$ is a feasible solution. Show that there is a feasible solution $\tilde{y}$ having the same value (that is, $c^T \tilde{y} = c^T \tilde{x}$) and having at most $(m+1)$ strictly positive components.

2. [20 + 10 marks] Consider the LPS: *Minimize* $(x_1 - x_2)$ subject to $x_1 + x_3 = 1$, $2x_2 + x_3 = 2$, $x_j \geq 0, j = 1, 2, 3$.

(i) Solve the problem using the simplex algorithm, starting from a basic feasible solution obtained from the first phase of the two-phase method.

(ii) Find all the extreme points, if any. What can you say about the feasible domain?

3. [15 + 10 marks] (i) Show that the system $Ax = b$ has a nonnegative solution *if and only if* there is no vector y satisfying $A^T y \leq 0$, $b^T y > 0$.

(ii) Show that the following problem is infeasible : *Minimize* $(c_1 x_1 + c_2 x_2 + c_3 x_3)$ subject to

$$x_1 + 3x_2 - 5x_3 = 2, \quad x_1 - 4x_2 - 7x_3 = 3, \quad x_j \geq 0, \ j = 1, 2, 3.$$

4. [12 marks] Suppose a primal LP problem and its dual are both feasible. Show that both the problems have optimal solutions. What can you say about the optimal values of the two problems?